Flow-Mach-Number-Variation-Induced Hysteresis in Steady Shock Wave Reflections

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Introduction

TWO shock-wave-reflection configurations, regular reflection (RR) and Mach reflection (MR), are possible in steady flows.[1] The RR and the MR consist of two and three shock waves, respectively. Two extreme angles of incidence are associated with the oblique reflection of a shock wave.[2] They are the von Neumann ϕ^N and the detachment ϕ^D angles ($\phi^D > \phi^N$). For a given flow Mach number, ϕ^N is the smallest angle of incidence for which an MR is theoretically possible, and ϕ^D is the largest angle of incidence for which an RR is theoretically possible. Consequently, an MR is impossible for $\phi < \phi^N$ and an RR is impossible for $\phi > \phi^D$. For incident angles in the range $\phi^N \le \phi \le \phi^D$ both RR and MR are theoretically possible. Consequently, the RR $\leftrightarrow$ MR transition could take place at any angle of incidence ϕ inside that range. The angle of incidence ϕ is determined by the flow Mach number M and wedge angle θ_w. Therefore, it is convenient to represent the detachment and von Neumann criteria in the (M, θ_w) plane, as is shown in Fig. 1, where θ_w^N and θ_w^D, the wedge angles that correspond to the mentioned criteria, are shown for a perfect gas with a specific heats ratio of $\gamma = 1.4$. The curves $\theta_w^N(M)$ and $\theta_w^D(M)$ divide the plane into three domains: a domain in which only RR is theoretically possible, a domain in which only MR is theoretically possible, and a domain, known as the dual-solution domain, in which both RR and MR are theoretically possible. Note that the curve corresponding to θ_w^N reaches a maximum value of $\theta_{w,\max}^N = 20.92$ deg at $M = 4.46$. At large values of M, the curves corresponding to both θ_w^N and θ_w^D approach asymptotic values equal to 17.96 and 32.02 deg, respectively.

Hysteresis in the RR $\leftrightarrow$ MR Transition

It was hypothesized in Ref. 3 that a hysteresis could exist in the RR $\leftrightarrow$ MR transition when the flow Mach number is kept constant and the wedge angle is changed in otherwise steady flows from a value of $\theta_w < \theta_w^N$ for which only an RR is theoretically possible to a value of $\theta_w > \theta_w^D$ for which only an MR is theoretically possible and then back to the initial value (along the path AA′A in Fig. 1). They hypothesized that the RR $\to$ MR transition would occur at the detachment criterion whereas the reversed MR $\to$ RR transition would take place at the von Neumann criterion. This hysteresis will be referred in the following as a wedge-angle variation-induced hysteresis.

An inspection of Fig. 1 indicates that the hysteresis can be also obtained in another way, keeping the wedge angle constant and changing the flow Mach number (flow Mach number variation-induced hysteresis). In this case there exist two possibilities. If an RR is established inside the dual-solution domain, for example, at point B, and then the flow Mach number is changed to follow the path BB′, the RR $\to$ MR transition should take place when the BB′ path crosses the $\theta_w^D(M)$ curve. However, on the reversed path from B′ to B, the reversed MR $\to$ RR transition is not required because an MR is also possible at point B. The BB′B path is not a full hysteresis loop although both RR and MR wave configurations can be observed for the same values of the wedge angle and the flow Mach number. On the other hand, if $\theta_w < \theta_{w,\max}^N$, the flow Mach number can be changed from a value for which only an RR is theoretically possible, for example, point C, to a value for which only an MR is theoretically possible, for example, point C′, and then back to the initial value crossing both $\theta_w^N(M)$ and $\theta_w^D(M)$ curves (see the path CC′C). Consequently, similar to the wedge-angle variation-induced hysteresis, a full hysteresis loop with a return to the initial shock wave configuration could be realized.

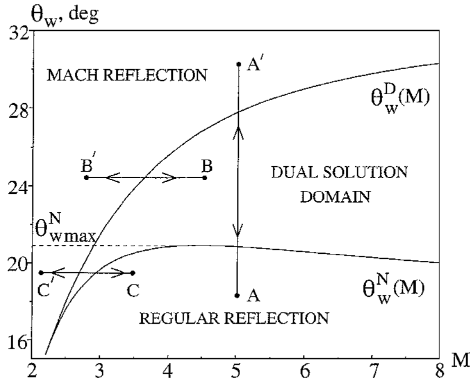

Fig. 1 Dependence of the wedge angles corresponding to the von Neumann and detachment criteria on the flow Mach number.

In Ref. 4, Hornung and Robinson failed to record experimentally the wedge-angle variation-induced hysteresis and adopted the conclusion that the RR is unstable in the dual-solution domain.[5] Based on these mentioned studies, it was believed until the late 1980s and early 1990s that a hysteresis does not exist in the shock-wave reflection process in steady flows. When a linear stability technique[6] was used and the principle of minimum entropy production[7] was applied, it was proved, in the early 1990s, that the RR is stable in most of the dual-solution domain.

The wedge-angle variation-induced hysteresis was recorded experimentally for the first time in Ref. 8, together with stable RR wave configurations inside the dual-solution domain. By using an Euler code it was illustrated[9] that the RR is indeed stable in the dual-solution domain. In Ref. 10, Ivanov et al. were the first to verify numerically, using a direct simulation Monte Carlo (DSMC) technique, the existence of the wedge-angle variation-induced hysteresis. Following their pioneering study, many investigators using different numerical codes verified the wedge-angle variation-induced hysteresis phenomenon numerically.

A similar wedge-angle variation-induced hysteresis, but with asymmetric wedges, was reported recently in Ref. 11. The suggested wedge-angle variation-induced hysteresis was verified experimentally. The experimental results showed excellent agreement with the analytical transition lines.

The described flow Mach number variation-induced hysteresis has not been revealed yet either experimentally or numerically. Because of the technical complexity in varying the flow Mach number in wind tunnels, verifying experimentally the flow Mach number variation-induced hysteresis is not feasible. Instead, the flow Mach number variation-induced hysteresis was verified in the present study by conducting a numerical simulation of the reflection process. The details and results of the numerical study are given next. (Note that the results of our numerical study of flow Mach number variation-induced hysteresis were partly presented in the plenary review Note presented in Ref. 12 and an independent investigation of this type of hysteresis was also reported in Ref. 13. The results in Ref. 13 are in close agreement with ours.)

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Numerical Method

The flow between two symmetrically spaced wedges was simulated. When it was assumed that viscous effects on the interaction of shock waves generated by the wedges were negligible, the unsteady Euler equations were solved. The perfect gas model with $\gamma = 1.4$ was used. The simulations were performed with a shock-capturing total variation diminishing (TVD) scheme. The fourth-order formula was utilized to reconstruct cell face values of the primitive variables (the density, the pressure, and the velocity components) from cell-averaged ones. Numerical fluxes were calculated by the Harten–Lax–van Leer–Einfeldt (HLLE) approximate Riemann solver, which is very robust for modeling high-speed flows. Time integration was accomplished by the third-order explicit TVD Runge–Kutta scheme. A more detailed description of the numerical techniques may be found in Ref. 14.

Because of the symmetry of the problem, the computations were performed only in one-half of the domain (Fig. 2). The length of the inclined section of the wedge surface is w, and the distance from the trailing edge of the wedge to the plane of symmetry is g. The ratio g/w was chosen to be 0.42. This value meets two conditions: The incident shock wave does not interact with the expansion fan emanating from the trailing edge of the wedge, and the reflected shock wave does not impinge on the wedge surface. The computational domain was divided approximately into 60,000 quadrilateral cells. A uniform supersonic flow was specified on the left (inflow) boundary of the domain and the zero-order extrapolation was employed on the right (outflow) boundary. The bottom boundary was treated as a plane of symmetry, and solid wall conditions were imposed on the top boundary.

The inflow Mach number was varied by changing the boundary conditions. The inflow pressure and density were kept constant while the velocity was changed so that an increment of the flow Mach number was equal to 0.05. As a result, weak disturbances were formed on the left boundary. They moved downstream and interacted with the shock reflection configuration. The solution was integrated in time until this unsteady transient process was completed, and the flow reached a steady state. After that, the flow Mach number was changed again.

To ensure the independence of the results on the grid resolution, some of the computations were repeated with finer grids (up to 160,000 cells). The agreement between the coarse and fine grid results was excellent.

Numerical Results

Both the $\theta_w > \theta^N_{w,\max}$ and $\theta_w < \theta^N_{w,\max}$ cases were considered. Typical results for the former are shown in Fig. 2. The wedge angle was kept constant during the simulation at $\theta_w = 27$ deg $> \theta^N_{w,\max} = 20.92$ deg while the flow Mach number was first decreased from 5 to 4.45 and then increased back to 5. Figure 2, frame 1, shows an RR, with $M = 5$, inside the dual-solution domain. As M was reduced, the detachment transition line beyond which an RR is theoretically impossible was reached at $M = 4.57$. The RR $\rightarrow$ MR transition took place when the Mach number was changed from 4.5 to 4.45 (Fig. 2, frames 3 and 4). Based on these two frames, the transition occurred at $M = 4.475 \pm 0.025$, in reasonable agreement with the theoretical value of $M = 4.57$. The existence of an RR slightly beyond the theoretical limit has been also observed in many numerical simulations of the wedge-angle variation-induced hysteresis and can be explained by the influence of numerical viscosity inherent in any shock-capturing code. Once MR was established, the flow Mach number was increased up to its initial value $M = 5$. Because theoretically an MR can exist for values of M inside the dual-solution domain, the reversed MR $\rightarrow$ RR transition did not take place at the detachment line. As a result, two different stable wave configurations, an RR and an MR, were obtained inside the dual-solution domain for identical flow conditions, that is, M and ϕ or θ_w. This can clearly be seen in Fig. 2 by comparing the pairs of frames 1 and 7, 2 and 6, and 3 and 5 in which the first frame is an RR and the second one is an MR, respectively.

The second series of simulations was performed at $\theta_w = 20.5$ deg $< \theta^N_{w,\max} = 20.92$ deg. For this value of θ_w, the Mach number values that correspond to the von Neumann criterion are 3.47 and 6.31, whereas that corresponding to the detachment criterion is 2.84. The Mach number was decreased from $M = 3.5$ to 2.8, and then increased up to the initial value. Some frames showing the sequence of events are given in Fig. 3. The RR $\rightarrow$ MR transition occurs between $M = 2.9$ and 2.8, in close agreement with the theoretical value, whereas the reverse, MR $\rightarrow$ RR transition, is observed between $M = 3.2$ and 3.3, that is, slightly earlier than that predicted theoretically. This disagreement can be attributed to the very small height of

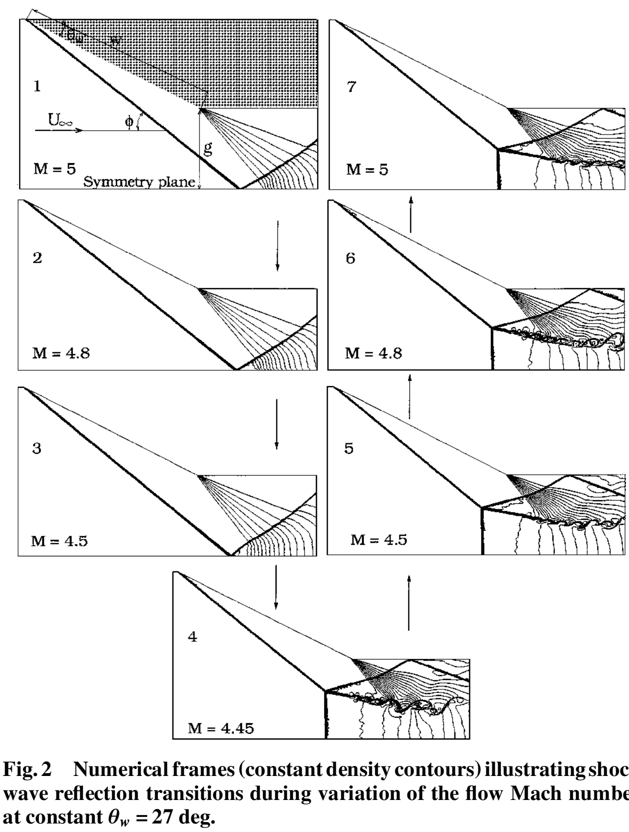

Fig. 2 Numerical frames (constant density contours) illustrating shock wave reflection transitions during variation of the flow Mach number at constant $\theta_w = 27$ deg.

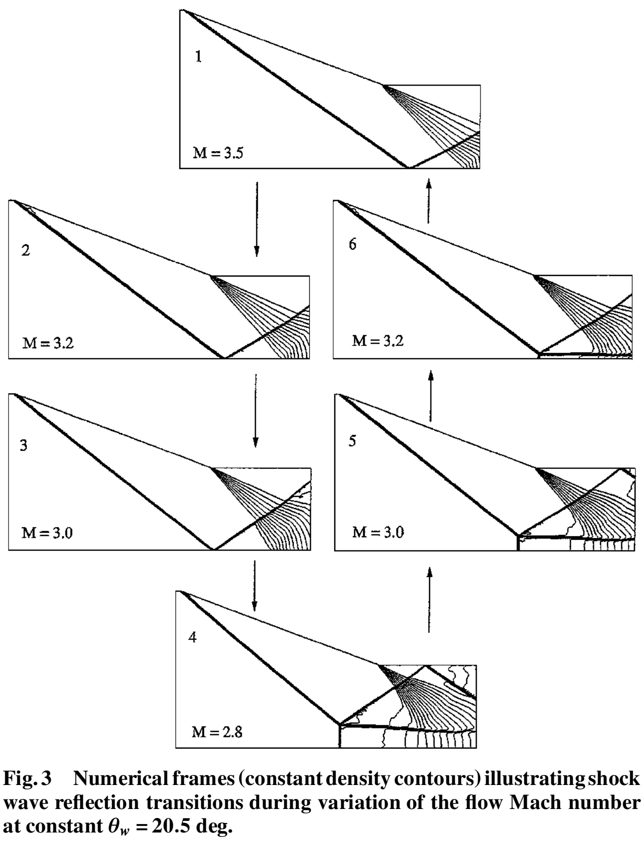

Fig. 3 Numerical frames (constant density contours) illustrating shock wave reflection transitions during variation of the flow Mach number at constant $\theta_w = 20.5$ deg.

the Mach stem near the von Neumann criterion, which makes its numerical resolution very difficult. Figure 3 demonstrates clearly the existence of the flow Mach number variation-induced hysteresis.

Conclusions

It was shown numerically that, in addition to the wedge-angle variation-induced hysteresis in the RR $\leftrightarrow$ MR transition, which was first illustrated numerically in Ref. 10, a flow Mach number variation-induced hysteresis is also possible in two-dimensional steady flows. For the latter, two different shock wave reflection configurations can be obtained at the same values of angle of incidence and flow Mach number, depending on the direction from which the flow Mach number was reached.

Because the investigated geometry resembles the geometry of supersonic intakes, the new hysteresis type that is reported in the present study can be relevant to flight performance at high supersonic speeds. The possible dependence of the flow pattern on the preceding maneuvers of an aircraft should be taken into account in designing intakes for perspective hypersonic vehicles.

Acknowledgment

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References

[1]Ben-Dor, G., *Shock Wave Reflection Phenomena*, Springer-Verlag, New York, 1991, Chap. 3.

[2]Von Neumann, J., "Oblique Reflection of Shocks," Navy Dept., Explosive Research Rept. 12, Bureau of Ordnance, Washington, DC, 1943 (reprinted in *Collected Works of J. von Neumann,* Vol. 6, Pergamon, Oxford, 1963, pp. 238–299).

[3]Hornung, H. G., Oertel, H., Jr., and Sandeman, R. J., "Transition to Mach Reflection of Shock Waves in Steady Flow With and Without Relaxation," *Journal of Fluid Mechanics*, Vol. 90, 1979, pp. 541–560.

[4]Hornung, H. G., and Robinson, M. L., "Transition from Regular to Mach Reflection of Shock Waves. Part 2. The Steady Flow Criterion," *Journal of Fluid Mechanics*, Vol. 123, 1982, pp. 155–164.

[5]Henderson, L. F., and Lozzi, A., "Further Experiments on Transition to Mach Reflection," *Journal of Fluid Mechanics,* Vol. 94, 1979, pp. 541–559.

[6]Teshukov, V. M., "On Stability of Regular Reflection of Shock Waves," *Prikladnaya Mekhanika i Technicheskaya Fizika (Applied Mechanics and Technical Physics)*, No. 2, 1989, pp. 26–33 (in Russian).

[7]Li, H., and Ben-Dor, G., "Application of the Principle of Minimum Entropy Production to Shock Wave Reflections. I. Steady Flows," *Journal Applied Physics*, Vol. 80, 1996, pp. 2027–2037.

[8]Chpoun, A., Passerel, D., Li, H., and Ben-Dor, G., "Reconsideration of the Oblique Shock Wave Reflection in Steady Flows. I. Experimental Investigation," *Journal of Fluid Mechanics*, Vol. 301, 1995, pp. 19–35.

[9]Vuillon, J., Zeitoun, D., and Ben-Dor, G., "Reconsideration of the Oblique Shock Wave Reflection in Steady Flows. I. Experimental Investigation," *Journal of Fluid Mechanics*, Vol. 301, 1995, pp. 37–50.

[10]Ivanov, M. S., Gimelshein, S. F., and Beylich, A. E., "Hysteresis Effect in Stationary Reflection of Shock Waves," *Physics of Fluids*, Vol. 7, 1995, pp. 685–687.

[11]Li, H., Chpoun, A., and Ben-Dor, G., "Analytical and Experimental Investigations of the Reflection of Asymmetric Shock Waves in Steady Flow," *Journal of Fluid Mechanics*, Vol. 390, 1999, pp.25–43.

[12]Ben-Dor, G., "Hysteresis Phenomena in Shock Wave Reflections in Steady Flows," *Proceedings of 22nd International Symposium on Shock Waves*, Vol. 1, edited by G. J. Ball, R. Hillier, and G. T. Roberts, Univ. of Southampton, Southampton, England, U.K., 1999, pp. 49–56.

[13]Onofri, M., and Nasuti, F., "Theoretical Considerations on Shock Reflections and Their Implications on the Evaluations of Air Intake Performance," *Proceedings of 22nd International Symposium on Shock Waves*, Vol. 2, edited by G. J. Ball, R. Hillier, and G. T. Roberts, Univ. of Southampton, Southampton, England, U.K., 1999, pp. 1285–1290.

[14]Khotyanovsky, D. V., Kudryavtsev, A. N., and Ivanov, M. S., "Numerical Study of Transition Between Steady Regular and Mach Reflection Caused by Free-Stream Perturbations," *Proceedings of 22nd International Symposium on Shock Waves*, Vol. 2, edited by G. J. Ball, R. Hillier, and G. T. Roberts, Univ. of Southampton, Southampton, England, U.K., 1999, pp. 1261–1266.

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Influence of External Caps on the Dynamic Behavior of Aerospace Cylindrical Vessels

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Introduction

A MORE complete structural numerical simulation model than the one utilized in a previous work[1] for the dynamic analysis of cylindrical tanks is necessary for the analysis of the dynamic behavior of vessel structures with axisymmetric caps at the ends, which can be applied to generic axisymmetric shells of revolution.

Flugge[2] introduced a simplified linear model for static and dynamic behavior of axisymmetric thin shells (also see Ref. 3). Narasimhan and Alwar[4] utilized the same model for a study of vibration of orthotropic spherical shells and introduced an interesting numerical procedure based on the Chebyshev–Galerkin spectral method for the evaluation of free vibration frequencies and modal shapes.

Hwang and Foster utilized a similar and simpler model for a study of the dynamic behavior of isotropic shallow spherical shells with a circular hole,[5] but the out-of-plane shear behavior and rotary inertia were not taken into account. They found a solution of the free vibrational frequency equation in terms of Bessel functions and modified Bessels functions.

Ozakca and Hinton[6] built a Mindlin[7]–Reisner[8] axisymmetric finite element model with the same kinematic relations, where the out-of-plane and rotary inertia effects with varying shell thickness are taken into account, for a free vibration analysis and optimization of axisymmetric shells of revolutions.

As in the previous work,[1] a simplified numerical model for the dynamic analysis of an orthotropic antisymmetric angle-ply laminated axisymmetric shell has been developed here. The considered structure is the same as the one dealt with by Mizusava and Kito,[9] who utilized the first-order shear deformation Sanders' shell theory to analyze the vibration behavior. An out-of-plane shear stress distribution along the thickness coordinate according the Mindlin[7]–Reisner[8] theory is imposed. The same kinematic relations utilized by the mentioned authors,[1–5] with appropriate approximations, have been employed to build this structural model.

A numerical procedure,[1,10–13] which lies between the Rayleigh–Ritz method (see Refs. 14 and 15) and the finite element method (FEM)[3,16,17] and which is obtained by combining the Ritz analysis with the variational principles,[18–20] has been applied to find the free frequencies and vibration modes.

We have seen in the whole cylindrical structure case that there are low-frequency (lf) vibrating modes, flexural and flexural–torsional (FT) modes, and high-frequency (hf) shear vibration modes, where the displacements can be neglected with respect to the rotations: These can be divided into shear–flexural (SF) modes, where the rotation ϕ_s is predominant with respect to the rotation ϕ_y, and shear–torsional (ST) modes, where, on the contrary, ϕ_s can be neglected with respect to ϕ_y.

The same vibration modes have been considered here, particularly with regard to the influence of the external axisymmetric caps on them. Finally, the dependence of both lf and hf on the winding angle on the cylindrical central part, which can be an important parameter in the design of the aerospace vehicles vessels, has been considered.

Mathematical Model

A vessel shell profile is considered and a reference system s, y, z_s (Fig. 1), where s and y are oriented along the tangent to the cap

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